1 A

$$\lim_{n\to+\infty} \frac{e^{2\beta n}}{3^n} \sqrt{\frac{(n+1)^{n+2\beta}}{n^n}} \frac{1}{(n+3)^\beta} =$$

$$= \lim_{n\to+\infty} \left(\frac{e^{2\beta}}{3}\right)^n \sqrt{\left(\frac{n+1}{n}\right)^n} \left(\frac{n+1}{n+3}\right)^\beta =$$

$$= \lim_{n\to+\infty} \left(\frac{e^{2\beta}}{3}\right)^n \sqrt{\left(1+\frac{1}{n}\right)^n} = \lim_{n\to+\infty} \left(\frac{e^{2\beta}}{3}\right)^n \cdot \sqrt{e}$$

Se $\frac{e^{2\beta}}{3} > 1 \iff \beta > \frac{1}{2}\log 3 \rightarrow \ell = +\infty$

se $\beta = \frac{1}{2}\log 3 \rightarrow \ell = \sqrt{e}$

se $0 < \beta < \frac{1}{2}\log 3 \rightarrow \ell = \emptyset$

B

$$\lim_{n\to+\infty} \frac{2^n}{e^{\alpha n}} \sqrt[3]{\frac{(n+2)^n}{(n+1)^{n+\alpha}}} \cdot n^{\alpha/3}$$

$$= \lim_{n\to+\infty} \left(\frac{2}{e^\alpha}\right)^n \sqrt[3]{\left(1+\frac{1}{n+1}\right)^n} \left(\frac{n}{n+1}\right)^{\alpha/3}$$

$$= \lim_{n\to+\infty} \left(\frac{2}{e^\alpha}\right)^n \cdot \sqrt[3]{e}$$

Se $\frac{2}{e^\alpha} > 1 \iff 0 < \alpha < \log 2 \rightarrow \ell = +\infty$

$\alpha = \log 2 \rightarrow \ell = \sqrt{e}$

$\alpha > \log 2 \rightarrow \ell = \emptyset$

$\boxed{2}$ (A)

$$\boxed{\lim_{x\to 0} \frac{e^{x-\frac{x^2}{2}} - \log(1+x^3+x) + \sin\left(x-\frac{x^2}{2}\right) - 1 - x + 11\tan\left(\frac{x^3}{6}\right)}{\sin(x^2+1)\left(\cos(x^2)-1\right)}}$$

$\boxed{N}$

$$e^{x-\frac{x^2}{2}} = \cancel{x} - \frac{x^2}{2} + \frac{1}{2}\left(x-\frac{x^2}{2}\right)^2 + \cancel{\frac{1}{6}\left(x-\frac{x^2}{2}\right)^3} + \frac{1}{24}x^4 + o(x^4)$$

$$-\log(1+x^3+x) = -x^3 - \cancel{x} + \frac{1}{2}(x^3+x)^2 - \frac{1}{3}(x^3+x)^3 + \frac{1}{4}x^4 + o(x^4)$$

$$\operatorname{sen}\left(x-\frac{x^2}{2}\right) = \cancel{x} - \frac{x^2}{2} \cancel{- \frac{1}{6}\left(x-\frac{x^2}{2}\right)^3} + o(x^4)$$

$$-x + 11\tan\left(\frac{x^3}{6}\right) = \cancel{-x} + \frac{11}{6}x^3 + o(x^4)$$

Sommando tutti i termini a numeratore si ha

$$\cancel{-\frac{x^2}{2}} + \frac{1}{2}x^2 - \frac{1}{2}x^3 + \frac{1}{8}x^4 + \frac{1}{24}x^4$$

$$-x^3 + \cancel{\frac{1}{2}x^2} + x^4 - \frac{1}{3}x^3 + \frac{1}{4}x^4$$

$$\cancel{-\frac{x^2}{2}} + \frac{11}{6}x^3 + o(x^4) =$$

$$= \cancel{\left(\frac{-3-6-2+11}{6}\right)}x^3 + \left(\frac{3+1+24+6}{24}\right)x^4 + o(x^4)$$

$$= \frac{34}{24}x^4 = \boxed{\frac{17}{12}x^4}$$

$\boxed{D}$: $\operatorname{sen}(x^2+1) \to \operatorname{sen} 1$ per $x \to 0$

$$\cos(x^2) - 1 = -\frac{1}{2}x^4 + o(x^4)$$

$$\Rightarrow \lim_{x\to 0} \frac{N}{D} = \lim_{x\to 0} \frac{\frac{17}{12}x^4 + o(x^4)}{(\operatorname{sen} 1)\cdot\left(-\frac{1}{2}x^4 + o(x^4)\right)} = \boxed{-\frac{17}{6\operatorname{sen} 1}}$$

(2) (3)

$$\lim_{x\to 0} \frac{\sin\left(x+\frac{x^2}{2}\right)+1-e^{x-x^3}+\frac{1}{2}\log\left(1+x+\frac{x^2}{2}\right)-\frac{7}{12}\operatorname{arctg} x^3-\frac{x}{2}}{\left(\sqrt{1-2x^4}-1\right)e^{x+1}}$$

$\boxed{N}$: $\sin\left(x+\frac{x^2}{2}\right)=\left(x+\frac{x^2}{2}\right)-\frac{1}{6}\left(x+\frac{x^2}{2}\right)^3+o(x^4)$

$1-e^{x-x^3}=-(x-x^3)-\frac{1}{2}(x-x^3)^2-\frac{1}{6}(x-x^3)^3-\frac{1}{24}x^4+o(x^4)$

$-\frac{7}{12}\operatorname{arctg} x^3-\frac{x}{2}=-\frac{7}{12}x^3+o(x^4)-\frac{x}{2}$

$\frac{1}{2}\log\left(1+x+\frac{x^2}{2}\right)=\frac{1}{2}\left(x+\frac{x^2}{2}\right)-\frac{1}{4}\left(x+\frac{x^2}{2}\right)^2+\frac{1}{6}\left(x+\frac{x^2}{2}\right)^3-\frac{1}{8}x^4+o(x^4)$

Sommando tutti i termini a numeratore si ha:

$N=\cancel{x}+\cancel{\frac{x^2}{2}}-\cancel{x}+\boxed{x^3}-\cancel{\frac{1}{2}x^2}+x^4\boxed{-\frac{1}{6}x^3}-\frac{1}{24}x^4$
$\boxed{-\frac{7}{12}x^3}-\cancel{\frac{x}{2}}+\cancel{\frac{1}{2}x}+\cancel{\frac{1}{4}x^2}-\cancel{\frac{1}{4}x^2}-\frac{x^4}{16}-\frac{1}{8}x^4+o(x^4)$
$\boxed{-\frac{x^3}{4}}$

$=\cancel{\left(\frac{+12-2-7-3}{12}\right)x^3}+\left(\overbrace{\frac{+24-1-3}{24}}^{\frac{10}{12}}-\frac{1}{16}\right)x^4+o(x^4)$

$=\left(\frac{+40-3}{48}\right)x^4+o(x^4)$

$=+\frac{37}{48}x^4+o(x^4)$

$D=\sqrt{1-2x^4}-1=\cancel{1}-\frac{1}{2}\cdot 2x^4\cancel{-1}+o(x^4)=$
$=-x^4+o(x^4)$

$e^{x+1}\to e$ per $x\to 0$

$\lim_{x\to 0}\frac{N}{D}=$
$\Rightarrow$ $\boxed{-\frac{37}{48e}}$

$$\int_0 \frac{1}{(2-e^{-x})^2}\,dx \qquad \Rightarrow x = -\log t \qquad \Rightarrow dx = -\frac{1}{t}\,dt$$

$$\Rightarrow -\int_1^{e^{-1}} \frac{1}{(2-t)^2 \cdot t}\,dt = \int_{e^{-1}}^{1} \frac{1}{(2-t)^2 \cdot t}\,dt$$

Cerco A, B, C tali che

$$\frac{1}{(2-t)^2 \cdot t} = \frac{A}{2-t} + \frac{B}{(2-t)^2} + \frac{C}{t}$$

$$\Rightarrow At(2-t) + Bt + C(2-t)^2 = 1$$

$$\Leftrightarrow -At^2 + 2At + Bt + 4C + t^2C - 4Ct = 1$$

$$\Leftrightarrow t^2(-A+C) + t(2A - B - 4C) + 2B + 4C = 1$$

$$\Leftrightarrow \begin{cases} -A + C = 0 \\ 2A + B - 4C = 0 \\ 4C = 1 \end{cases} \qquad \begin{cases} A = C \\ B = 2C \\ C = \frac{1}{4} \end{cases} \qquad \begin{cases} A = \frac{1}{4} \\ B = +\frac{1}{2} \\ C = \frac{1}{4} \end{cases}$$

$$\Rightarrow \int_{1/e}^{1} \left(\frac{1}{4}\,\frac{1}{2-t} + \frac{1}{2}\,\frac{1}{(2-t)^2} + \frac{1}{4t} \right) dt =$$

$$= \left[\frac{1}{4}\log|2-t| - \frac{1}{2}\,\frac{1}{(2-t)} + \frac{1}{4}\log|t| \right]_{1/e}^{1}$$

= finire i calcoli...

l'altro è analogo

④ $f(x) = \sqrt{\dfrac{x^2-2}{|x-3|}}$ ⑤

DOMINIO: $\begin{cases} x^2-2 \geq 0 \\ x \neq 3 \end{cases} \Rightarrow \begin{cases} x \leq -\sqrt{2} \vee x \geq \sqrt{2} \\ x \neq 3 \end{cases}$

$$D = (-\infty, -\sqrt{2}] \cup [\sqrt{2}, 3) \cup (3, +\infty)$$

LIMITI:

- $\lim\limits_{x \to \pm\infty} f(x) = +\infty$
- $\lim\limits_{x \to 3^\pm} f(x) = +\infty$

DERIVATA:

$$f'(x) = \frac{1}{2\sqrt{\frac{x^2-2}{|x-3|}}} \cdot \frac{2x|x-3| - \operatorname{sgn}(x-3)(x^2-2)}{|x-3|^2}$$

$$= \frac{\operatorname{sgn}(x-3)}{2\sqrt{\frac{x^2-2}{|x-3|}} \cdot |x-3|^2} \cdot \left(2x(x-3) - x^2 + 2\right)$$

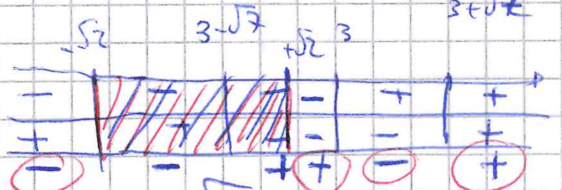
$$= \frac{\operatorname{sgn}(x-3)}{\underbrace{2\sqrt{\frac{x^2-2}{|x-3|}}\,|x-3|^2}_{>0}} \cdot \underbrace{\left(2x^2 - 6x - x^2 + 2\right)}_{x^2-6x+2}$$

$\forall x \in D \setminus \{-\sqrt{2}, +\sqrt{2}\}$

STUDIO IL SEGNO di f':

- $\operatorname{sgn}(x-3) > 0 \iff x > 3$
- $x^2 - 6x + 2 \geq 0 \iff x \leq 3-\sqrt{7} \vee x \geq 3+\sqrt{7}$

$x_{1,2} = 3 \pm \sqrt{9-2}$
$= 3 \pm \sqrt{7}$ $\Rightarrow$

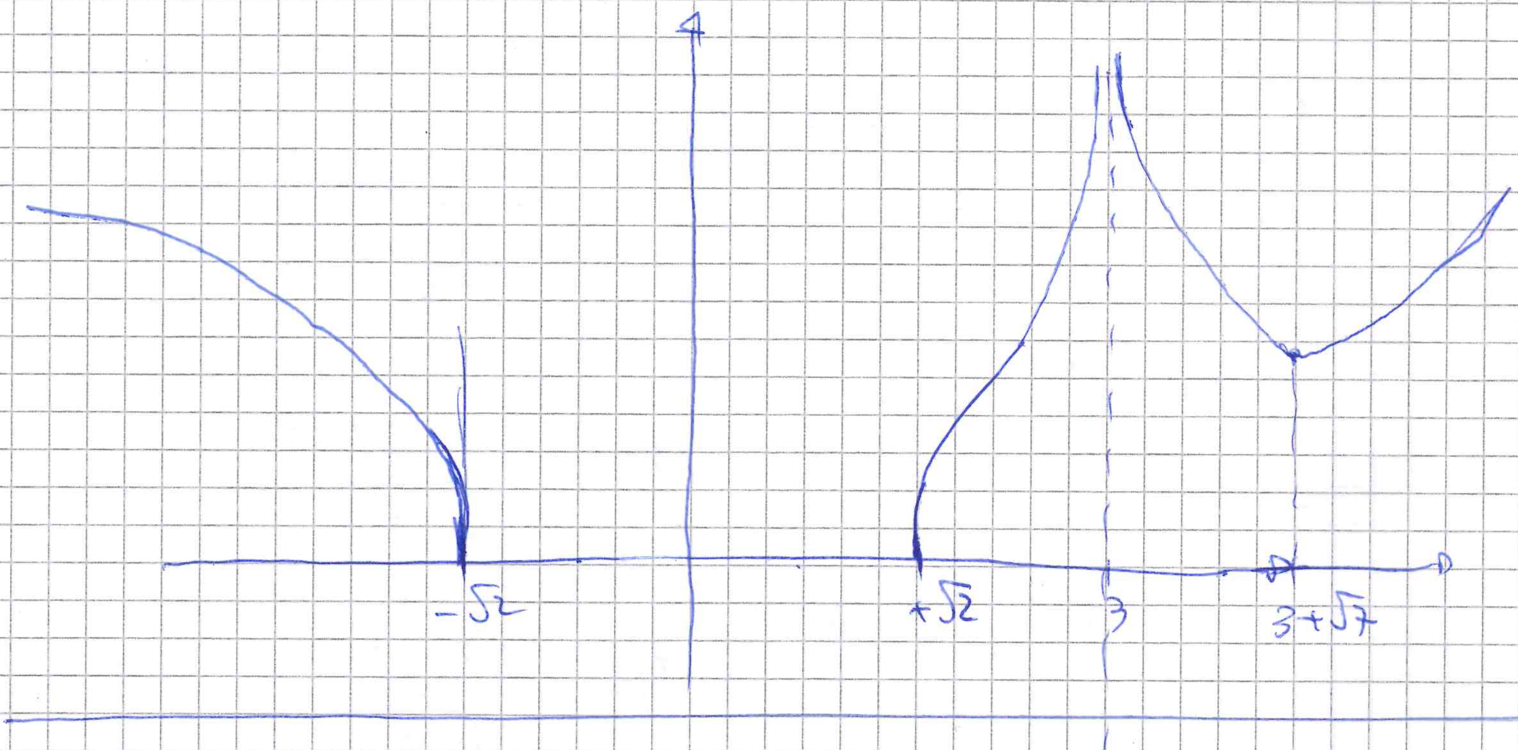


- $f \searrow$ in $(-\infty, +\sqrt{2}] \cup (3, 3+\sqrt{7}]$
- $f \nearrow$ in $(\sqrt{2}, 3) \cup (3+\sqrt{7}, +\infty)$

- $3+\sqrt{7}$ PUNTO di MINIMO LOCALE
- $\pm\sqrt{2}$ PUNTI di MINIMO ASSOLUTO

PUNTI di NON DERIVABILITÀ

$$\lim_{x \to \pm\sqrt{2}} f'(x) = +\infty \rightarrow \pm\sqrt{2}$$ PUNTI DI NON DERIVABILITÀ (la tangente è verticale)



(5) (A) $f(x) = e^{\sin(x^2+7)} \cdot \log(\cos x + 1)$

$$f'(x) = 2x\cos(x^2+7)e^{\sin(x^2+7)} \cdot \log(\cos x+1) + e^{\sin(x^2+7)} \cdot \frac{(-\operatorname{sen} x)}{\cos x + 1}$$

(B) $f(x) = \log(x + e^{\operatorname{sen} x}) \cdot e^{\tan x}$

$$f'(x) = \frac{1 + \cos x\, e^{\operatorname{sen} x}}{x + e^{\operatorname{sen} x}} \cdot e^{\tan x} + \log(x + e^{\operatorname{sen} x}) \frac{e^{\tan x}}{\cos^2 x}$$